Hale School

Year 11 Semester 1 Examination, 2015

Write your name below:

MARKING KEY

Mathematics Methods

Teacher:_____

Section Two: Calculator-assumed

Booklet 3 of 3

TIME ALLOWED FOR THIS SECTION

Reading time before commencing:	Ten minutes
Working time for paper:	One hundred minutes

MATERIAL REQUIRED/RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SUPERVISOR
TWO Question/Answer booklets for Section Two – complete BOTH.
Formula Sheet (retained from Section One).

TO BE PROVIDED BY THE CANDIDATE

Standard Items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special Items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and calculators approved for use.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. Please check carefully, and if you have any unauthorised material with you, hand it to the supervisor **BEFORE** reading any further.

STRUCTURE OF THIS PAPER

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	9	9	50	50	37
Section Two: Calculator-assumed	15	15	100	85	63
				Total	100

INSTRUCTIONS TO CANDIDATES

1. Write your answers in this Question/Answer Booklet.

2. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

3. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

4. Fill in the number of the question that you are continuing to answer at the top of the page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

Section Two: Calculator Assumed 85 marks (63%)
This section has 15 questions. Answer all questions. Write your answers in the spaces provided.
Working time: 100 minutes

Question 18 **7 marks**

In this situation, the Universal set is the set of **positive integers less than or equal to 15** and contains the subsets:

$$A = set\ of\ even\ numbers$$

$$B = set\ of\ prime\ numbers$$

$$C = \{2, 5, 7, 10, 12, 13\}$$

a. List the elements of set $A \cup C$. (1 mark)

$\{2, 4, 5, 6, 7, 8, 10, 12, 13, 14\}$

✓Lists all elements

b. Determine $|\overline{C}|$ (1 mark)

9

✓Calculates correctly

c. Determine $n(A \cap B)$ (1 mark)

1

✓Calculates correctly

d. Is the statement $5 \in A$ true or false? Justify your answer. (2 marks)

False, 5 is not an element of A (5 is odd).

✓States False
✓Explains meaning of $\in$

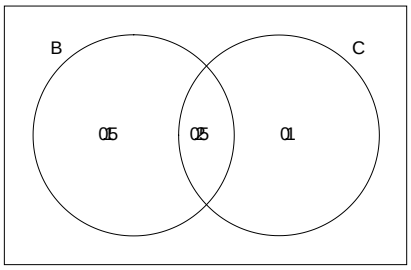
e. Is the statement $C \subset (A \cup B)$ true or false? Justify your answer. (2 marks)

True, all elements of C are in A or B.

✓States True
✓Explains meaning of $\subset$

Question 19 **5 marks**

The Venn diagram below shows the probabilities for sets B and C.



a. If a number from U is chosen at random, determine: (3 marks)

i. $P(B)$ 0.4

✓Calculates correctly

ii. $P(\overline{B \cup C})$

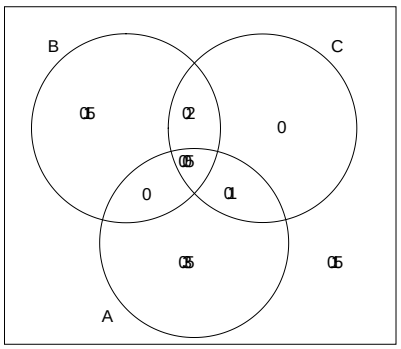
0.5

✓Calculates correctly

iii. $P(C|B)$ $\frac{0.25}{0.4} = \frac{5}{8}$

✓Calculates correctly

Set A is added to the Venn diagram as shown below.



b. If a number from U is chosen at random, determine: (2 marks)

i. $P(A \cap B \cap C)$

0.05

✓Calculates correctly

ii. $P(A \cup B \cup C)$

✓Calculates correctly

0.85

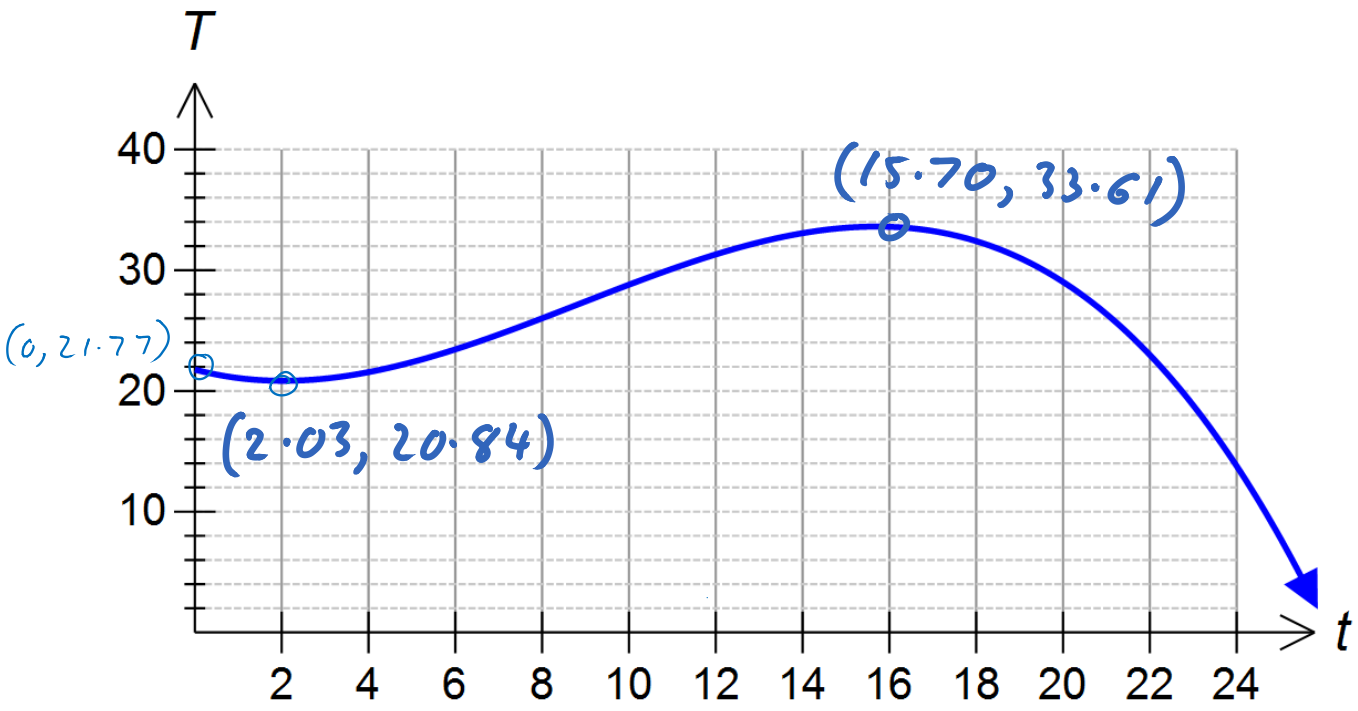
Question 20 **5 marks**

Hanna was using her calculator to determine a function that modelled the temperature T (in $°C$) on one day last summer. Her temperature function was

$$T = -0.01t^3 + 0.266t^2 - 0.957t + 21.77 .$$

The function applied from midnight ($t = 0$) to midnight ($t = 24$).

a. The graph of the temperature function for the 24 hour period is shown below. Label the coordinates of all key features on the graph, correct to 2 decimal places. (3 marks)



✓✓✓One mark per key point labelled with coordinates

(-1 mark for not rounding to 2dp correctly, -1 mark for only estimating coordinates from graph)

b. What did Hanna's function suggest the temperature was at 5:00 pm? (2 marks)
Answer accurately to 3 decimal places.

$t = 17$

$T = 33.245\ °C$

✓Determine $t = 0$

✓ Calculates T

(award 1 mark for t=5, T=22.385)

Question 21 **7 marks**

a. The probabilities for two events A and B are found to be $P(A) = 0.35$, $P(B) = 0.2$ and $P(A|B) = 0.6$. Determine: (4 marks)

i. $P(A \cap B)$ $= 0.6 \times 0.2 = 0.12$

✓Applies appropriate method (diagram, multiplication rule etc)
✓Calculates correctly

ii. $P(A \cup B)$

$= 0.35 + 0.2 - 0.12$

$= 0.43$

✓Applies appropriate method (diagram, addition rule etc)
✓Calculates correctly

b. Two events, D and E, are defined such that $P(D) = 0.6$, $P(D \cap E) = x$ and $P(\overline{D} \cap E) = 0.1$.

Determine x if sets *D* and *E* are independent. (3 marks)

$P(E) = 0.1 + x$

$P(D) \times P(E) = P(D \cap E)$

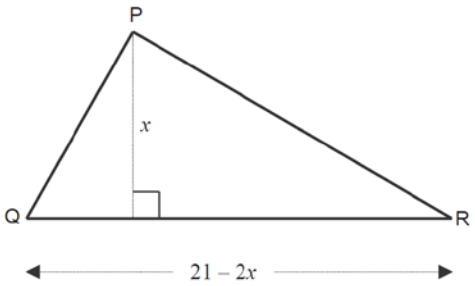
$0.6(0.1 + x) = x$

Solve: $x = 0.15$

✓Determines P(E) in terms of x
✓Applies test for independence
✓ Solves for x

Question 22 **5 marks**

Triangle PQR has a base of $(21 - 2x)\ cm$ and perpendicular height of $x\ cm$, as shown in the triangle below.

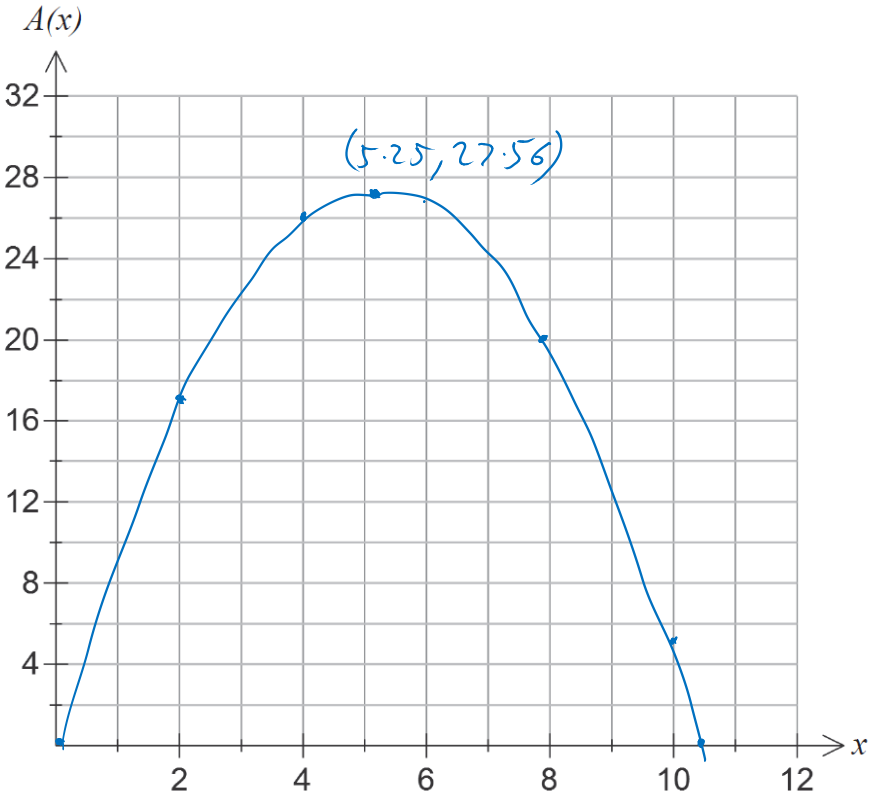


a. Determine an expression for the area function of the triangle in terms of x. (1 mark)

$\frac{1}{2}x(21-2x)$

✓ States correct expression

b. Sketch the area function below. Clearly label any intercepts or turning points. (3 marks)



✓ Identifies location of x-intercepts
✓ Identifies location of turning point
✓Draws neat, accurate parabola

c. What is the maximum possible area of triangle PQR? (1 mark)

$27.56\ cm^2$

✓ Determines maximum using graph or equation

Question 23 **6 marks**

a. Triangle XYZ has side lengths $x = 6\ cm$, $y = 10\ cm$ and an area of $15\sqrt{3}\ cm^2$. Determine the size of $\angle XZY$. (3 marks)

$15\sqrt{3} = \frac{1}{2}(6)(10)\sin Z$

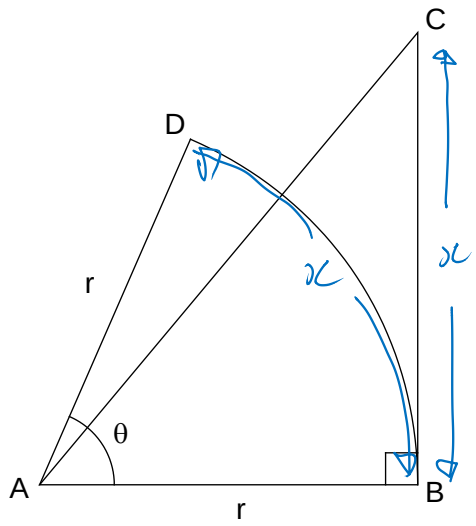
$Z = 60^\circ$ or 120°

✓ Substitutes values into area rule correctly

✓ Solves for one angle

✓ Determines second solution (radians also acceptable)

b. In the figure shown below, the arc BD is centred at A and has the same length as tangent segment BC. Show that the sector ABD will have the same area as the triangle ABC. (3 marks)



Sector Area $ABD = \frac{1}{2}r^2\theta$ (1)

Area $\triangle ABC = \frac{1}{2}bh = \frac{1}{2}rx$ (2)

Arc length: $x = r\theta$ (3)

Sub (3) → (1) $\text{Area}(ABD) = \frac{1}{2}r(r\theta)$

$= \frac{1}{2}rx$

$= \text{Area}\triangle ABC$

✓ States area of sector DAB in terms of given information

✓ States area of triangle CAB in terms of given information

✓ Shows that the areas are equivalent

Question 24 **7 marks**

The equation $x^2 + y^2 - 2ky - 2y + k^2 + 2k + 1 = 25$ represents a circle.

a. If the circle has a radius of $5\ cm$, state the coordinates of the centre of the circle in terms of k. (3 marks)

$x^2 + (y-k-1)^2 = 25$

✓ Factorises expression

Centre is

$(0, k+1)$

✓ Identify x-coordinate as zero

✓ Identify y-coordinate based on factorisation

b. If $k = 3$, find the coordinates of the point(s) where the circle intersects the line $y = 2x$. (4 marks)

$k=3 \Rightarrow x^2 + (y-4)^2 = 25$

✓Substitutes k into equation from question or part a

Sub $y = 2x$ into $x^2 + (y-4)^2 = 25$

✓Uses appropriate method to solve (graph on calc, substitution etc)

$x = -0.488$ and $x = 3.688$

$y = -0.976$ $y = 7.376$

✓ States both x-coordinates

✓ States corresponding y-coordinates

$(-0.49, -0.98)$ and $(3.69, 7.38)$

END OF BOOKLET 3